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An empirical stream power formulation for knickpoint retreat in Appalachian Plateau fluviokarst

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Summary Hydrologically abandoned caves on tributaries of the Upper Cumberland River, Tennessee, USA record a wave of river incision that advanced up the drainage basin in the Late Pliocene and Early Pleistocene. Geomorphic and geologic evidence suggests that incision occurred as a migrating knickpoint generated by sudden base-level lowering. The passage of a knickpoint up the Cumberland River tributaries was modeled as a perturbation to steady-state incision according to the stream power law $E = kQ^mS^n$ and tested using dated incision events recorded in cave sediments. Knickpoint migration rates generated by this model were 0.1–0.18 m/yr over the entire stream network, and 4.0 m/yr for the mainstem Cumberland River channel. The ratio $m/n = 0.79$ was consistent with previously published parameters; however, the values of $m = 1.91$ and $n = 2.39$ were much higher than those reported in previous field studies. These results suggest the stream power model may be used to model knickpoint migration in the study area, provided values for the constants m and n are larger. This may be due to the influence of fluviokarst, where surface drainage is interrupted due to diversion into the underlying karst aquifer. Field measurements of channel and basin geometry in fluviokarstic tributaries to the Upper Cumberland River show (1) a stronger variance between channel slope and discharge; (2) a nonlinear relationship between discharge and drainage area; and (3) stream width to be nearly invariant, as opposed to non-karst watersheds. Because the stream power model relies heavily on the substitution of discharge for drainage area, the behavior of channel incision and knickpoint migration in fluviokarst may differ substantially from that of non-karst channels.

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Introduction

River incision into bedrock is the visible expression of landscape response to regional, continental, or global changes in base-level due to climate, eustasy, or tectonics. Rapid base-level lowering is often considered on both theoretical and empirical grounds to be transmitted upstream via knickpoint propagation, or migration, following the stream power law. The rate of knickpoint migration is therefore a key determinant of landscape response to sudden base-level lowering. The Upper Cumberland River (UCR) of Tennessee and Kentucky (Fig. 1) displays several knickpoints along its mainstem and in all major tributaries in the form of waterfalls and steep reaches perched on resistant rock units (Fig. 2). In this study, we test the simplest form of the stream power model with the timing of a pulse of river incision that migrated up fluviokarst tributaries of the UCR beginning more than 2 million years ago (Anthony and Granger, 2006a,b). This date coincides with a major marine regression at 2.4 Ma (Galloway et al., 2000), which may be responsible for initiating a pulse of incision up the Mississippi River, to the Ohio River, and into the Cumberland River basin.

The timing of knickpoint migration up the Cumberland River and its tributaries is recorded by the hydrologic abandonment of cave passages that once discharged onto the river as springs (Anthony and Granger, 2004). In areas where large, horizontal cave passages develop in step with changes in regional base level, the elevation of abandoned cave passages represents the paleoelevation of the modern rivers, and the timing of passage abandonment may be dated by measuring cosmogenic nuclides in buried cave sediments. This type of hydrogeologic setting in the UCR basin offers distinct advantages for testing the stream power model for knickpoint migration. The major advantage is the reconstruction of incision history from cave morphology and burial dating of cave sediments (Granger et al., 2001; Anthony and Granger, 2004, 2006a,b). Among the disadvantages of working in fluviokarst watersheds include: a highly nonlinear discharge/drainage area relationship due to the two-component nature of fluviokarst baseflow; nearly invariant stream width along the channel, as opposed to non-karst watersheds; and convex rather than concave stream profiles (see White and White, 1989; Ford and Williams, 1989 for general discussions of fluviokarst hydrology).

The stream power model and knickpoint migration

Investigations of river incision in bedrock channels have utilized a model that links incision E to stream power (based on shear stress), with stream power related to discharge Q and stream gradient S by a power function (Howard and Kerby, 1983; Seidl and Dietrich, 1992; Howard et al., 1994; Sklar and Dietrich, 2001). Drainage area A usually appears as an easily-measured proxy for channel discharge (Leopold and Miller, 1956; Leopold et al., 1964). Additional factors such as rock resistance and climate are folded into the coefficient of erosion k. This model is referred to as the ''stream power law,'' and may be written in its simplest form as:

$$E = kA^mS^n \tag{1}$$

where m, n, and k are positive constants. The stream power model is appealing partly because of its simplicity but also because it can account for river characteristics and behavior such as profile concavity, knickpoint migration, and incision rates through time (Whipple et al., 2000).

Parameters in the general stream power model vary widely in terms of actual values in field studies where the model is applied (see Whipple and Tucker, 1999 for discus-

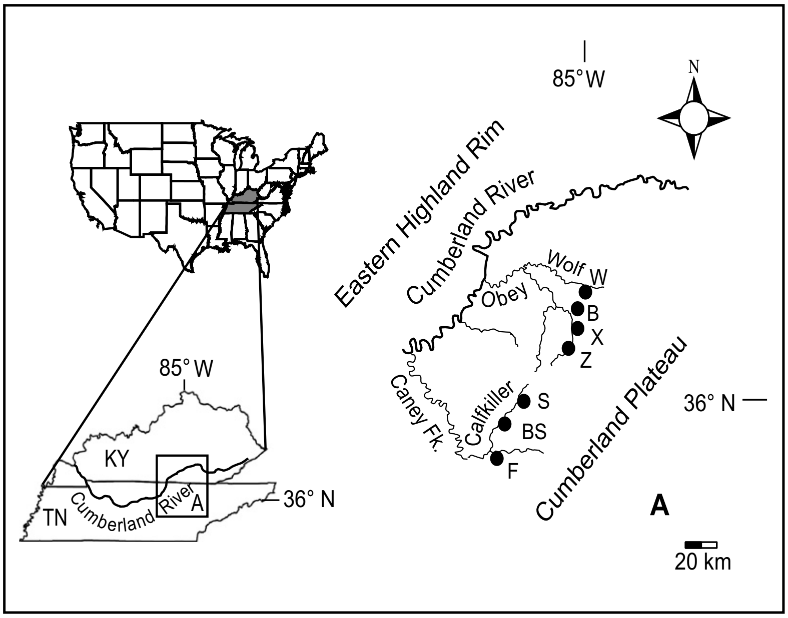

Figure 1 Location of Upper Cumberland River basin in Kentucky and Tennessee, USA. Caves on western margin of Cumberland Plateau drain into fluviokarst tributaries of the Cumberland River. F — Foxhole Cave; BS — Blue Spring Cave; S — Skagnasty Cave; W — Wolf River Cave; B — Buffalo Cave; X — Xanadu Cave; Z — Zarathustra's Cave. (See Anthony and Granger, 2004 for cave descriptions.)

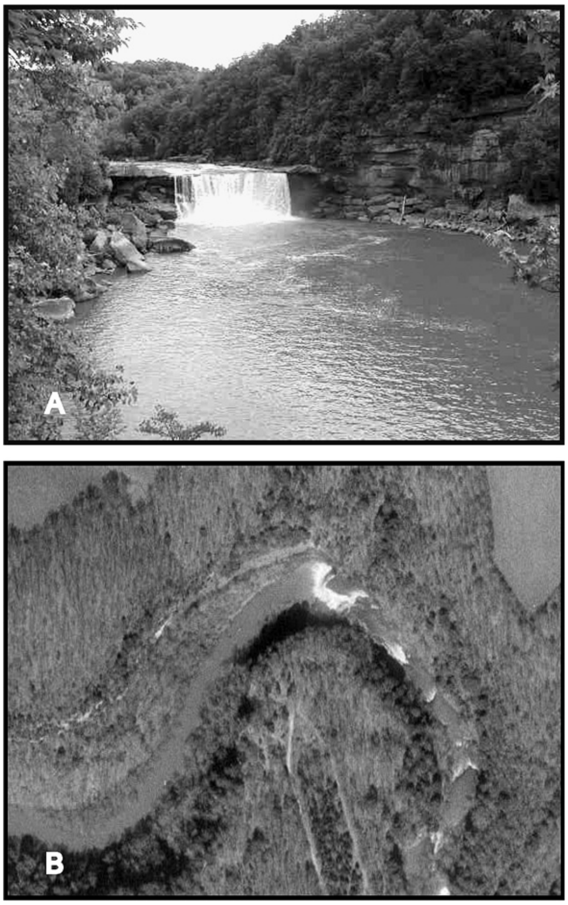

Figure 2 (A) Cumberland Falls, a 23-m waterfall on the Cumberland River; (B) air photo of Burgess Falls (40 m) and rapids on a tributary of the Cumberland River. Multiple knickpoints found on each tributary of the Upper Cumberland River suggest incision by knickpoint migration.

sion). Depending on how the stream power law is formulated, constants m and n are given as: $m = 0.45$ and $n = 0.7$, and $0.35 \leqslant m/n \leqslant 0.6$ for bedrock incision proportional to bed shear stress (Howard and Kerby, 1983); $m/n = 1.0$ for mainstem and tributary junctions (Seidl and Dietrich, 1992); $m = n = 1$ (Seidl et al., 1994) for the case of sudden base-level lowering; and $m = 0$–0.5 and $n = 0$–2 (Stock and Montgomery, 1999) for rocks of various channel gradients. The diversity of values underscores the need for additional field measurements with which to test the stream power model.

Many studies of incision by knickpoint migration have utilized the perturbation solution to the stream power law given by Rosenbloom and Anderson (1994); Whipple and Tucker (1999) for a knickpoint initiated by a sudden drop in base level. In that solution, the headward migration speed (celerity) c_e of a knickpoint may be modeled as a function of the stream power law (1) following:

$$c_e = -nkA^mS^{n-1} \qquad (2)$$

where a migrating knickpoint will slow at an ever decreasing rate. Other studies of river incision have assumed that incision by knickpoint migration is proportional to channel discharge raised to some power m (Howard and Kerby, 1983; Seidl and Dietrich, 1992).

In three recent studies, knickpoints created at known times in streams of different drainage areas have examined knickpoint migration in terms of channel discharge. In the first study, Hayakawa and Matsukura (2003) considered waterfall retreat rates in Japan for nine waterfalls with records extending from 50 to 6150 years and recession distances from 6.4 to 200 m. Discharge was modeled as the area of the watershed multiplied by mean annual precipitation. Regression of the data with discharge alone was able to account for less than 50% of the variance, unless one sample is discarded as an outlier. Then, the significance of the fit accounts for 82% of the variance with $m = 1.13$–1.16 (recalculated by Bishop et al., 2005).

A second recent study of knickpoint retreat achieved similar results over a somewhat longer timescale. Bishop et al. (2005) measured the positions of knickpoints on the eastern coast of Scotland, where a marine bench attests to uplift of the coast at approximately 14 ky ago. Knickpoints were identified on 14 different streams, and both drainage area and distance from the coast were noted. A regression of present-day drainage area versus distance from the coast yields a highly significant power-law relationship, where $m = 1.26$ and accounts for 92% of the variance of the data set. As in the data of Hayakawa and Matsukura (2003), the exponent on drainage area is slightly higher than one.

A third recent field study provides a more comprehensive analysis of a single large watershed. Crosby and Whipple (2005) analyzed 236 separate knickpoints in the Waipaoa River watershed, New Zealand. These knickpoints can be attributed to a single base-level fall at the mouth of the river approximately 18 ky ago. Crosby and Whipple found that the knickpoint data could best be explained using a simple power law whereby knickpoint speed increases with area raised by $m = 1.125$. In each of these cases, the rate of knickpoint migration is nearly linear with respect to drainage area. This is not entirely unexpected, since Gardner (1983); Seidl et al. (1994) realized that under the stream power law, $m = n = 1$ is required for knickpoints to migrate upstream while retaining their form.

Geomorphology and hydrogeology of the study area

The Cumberland River of Kentucky and Tennessee (Fig. 1) originates on the western flanks of the Appalachian Mountains and flows westward across the Appalachian and Interior Low Plateaus before joining with the Ohio River. The Cumberland Plateau at 550–610 m above sea level (m ASL) is capped with massive subhorizontal Pennsylvanian-age sandstone units underlain by Mississippian-age carbonate units. The carbonates extend westward from beneath the margin of the Cumberland Plateau and form the rolling karst topography of the Eastern Highland Rim (275–350 m ASL). A steep-walled gorge funnels the Cumberland River off of the Cumberland Plateau and onto the Eastern Highland Rim, where its channel is characterized thereafter by deeply incised meanders that wind across the landscape (Fenneman, 1938; Thornbury, 1965). Waterfalls and rapids (Fig. 2) are found along the Upper Cumberland River mainstem and every major tributary, including the Caney Fork and the Obey River (Fig. 1). These tributaries originate on the clastic rocks of the Cumberland Plateau and form steep, reentrant valleys on the plateau's western margin, exposing the limestone beneath. Both the Caney Fork and the Obey River are fluviokarstic in their upstream reaches.

Fluviokarst is unique topography and hydrology formed by the combination of fluvial and karst processes in areas where both soluble and insoluble rocks outcrop in the same drainage basin (White, 1988). In many fluviokarst regions, larger regional rivers, such as the Cumberland River, maintain their surface courses while at the same time are fed by intermittent tributaries. The drainage in fluviokarst is generally interrupted due to easy diversion into the underlying karst aquifer through fractures, sinkholes, and sinking streams. Surface drainage in fluviokarst is ephemeral (Fig. 3), and stream channels are filled only during large rain events and wet-weather months of the year when subsurface conduits can no longer store and transmit the input.

In the southeastern United States, average annual discharge (Q) for surface streams scales linearly with drainage area A, allowing drainage area to be used in place of discharge (Schumm, 1956; Hack, 1957; Brush, 1961). Base-flow discharge in karst regions is also shown to be directly related to drainage area (Hess et al., 1989; Hess and White, 1989; Quinlan and Ray, 1995). However, this relation differs for fluviokarst reaches, where discharge is measured using two components; a base-flow discharge measured at karst springs when surface channels are dry, and a high-flow discharge at karst springs combined with channel flow during storm events (Hess et al., 1989).

Figure 3 The East Fork-Obey River effectively demonstrates the nature of fluviokarst. (A) Flood stage in November, 2004; (B) Typical dry streambed in summer, 1981 (bridge in photo removed in mid-1980s).

In the study area, fluviokarst reaches have some component of discharge directed underground, varying from a small portion to the entire discharge. Low-order streams originating on the upland surface of the Cumberland Plateau flow down the steep escarpment of the western margin and sink at the sandstone/limestone contact (Fig. 4), where they form conduits (caves) following the hydraulic gradient to the local water table (Crawford, 1984). Underground streams emerge at the base of the escarpment as springs on tributaries to the Cumberland River. During periods of regional base level stability, large horizontal passages form at or near the local water table (Fig. 4) (Palmer, 1987, 1991), which is itself controlled by the elevation (position) of the Cumberland River. Sudden lowering of the water table by changes in the position of the Cumberland River initiates a rapid response underground, as the cave stream downcuts narrow canyons leading to the new water table and abandons upper-level cave passage (White and White, 1983). Hydrologically inactive (abandoned) cave passages in the Upper Cumberland River basin are thus related to former positions of the water table in the same way that river terraces are related, and abandonment marks the onset of regional river incision (Anthony and Granger, 2004).

Burial dating of cave sediments using cosmogenic nuclides

Fluvial sediments carried underground by sinking streams in the study area are easily identified in abandoned cave passages, and may remain undisturbed for millions of years (Anthony and Granger, 2004). These sediments are equivalent to fluvial deposits mantling strath terraces (Granger et al., 1997, 2001). Absolute ages of cave sediments determined by measurement of cosmogenic aluminum-26 (^{26}Al) and beryllium-10 (^{10}Be) are interpreted as the minimum age of active sediment transport by underground streams, and as the onset of the incision event that lowered the water table and abandoned the cave passage (Anthony and Granger, 2006a,b).

Burial dating is based on the production and radioactive decay of ^{26}Al (radioactive meanlife $\tau_{26} = 1.02 \pm 0.04$ Ma) and ^{10}Be ($\tau_{10} = 1.93 \pm 0.09$ Ma) in quartz crystals exposed to cosmic radiation near the surface prior to deposition underground by sinking streams. Quartz crystals in the sandstone caprock of the Cumberland Plateau accumulate these nuclides at known production rates by exposure to secondary cosmic-ray nucleons and muons (Lal and Peters, 1967; Lal, 1991). Deposition in caves effectively shields from further production; however, radioactive decay continues with ^{26}Al decaying at a faster rate than ^{10}Be. Measurement of the ''leftover'' or inherited ratio of nuclides in cave sediments allows for determination of burial age by iterative solution of equations for burial time, preburial erosion rate, and pre-

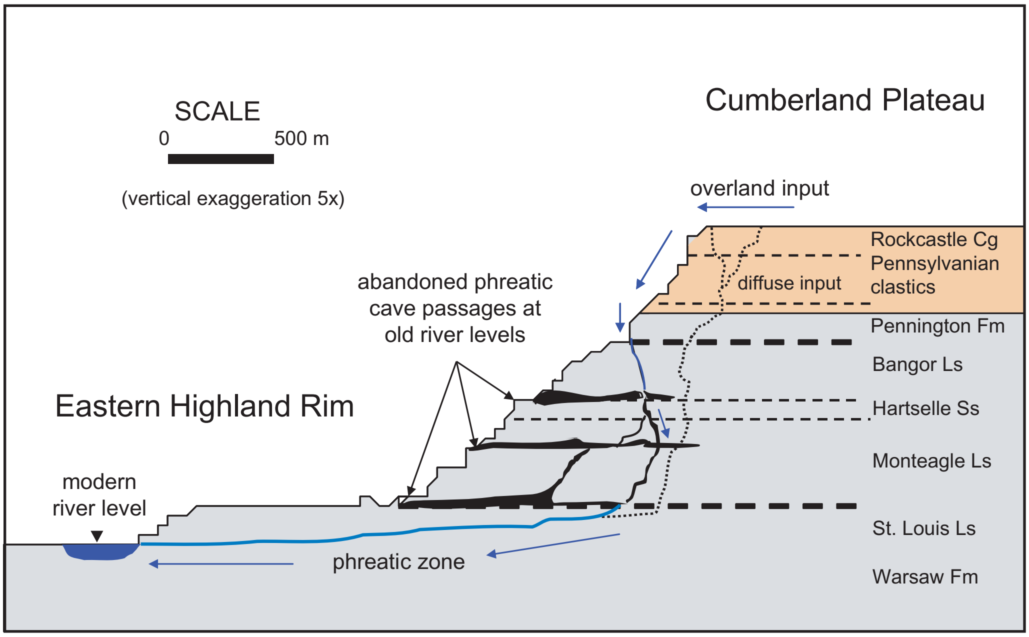

Figure 4 Schematic diagram of the western margin of the Cumberland Plateau and its cave-forming hydrogeology (after Crawford, 1984). The development of large, horizontal passages in multilevel caves is related to the position of the Cumberland River. Fluvial sediments in abandoned cave passages within the Monteagle Limestone are used to date the onset of river incision in this study.

burial concentrations of nuclides (see Granger et al., 1997, 2001 for equations).

Sediment samples were collected from seven caves in the Upper Cumberland River basin in passages located at similar elevations above the modern river level (Table 1). The caves were selected based on: (1) one or more abandoned passages, or levels, of large cross-sectional area; (2) extensive horizontal development; and (3) in-place open channel sediment deposits with no remobilization of sediments from upper levels or surface. An important assumption in burial dating is that the sediments have one period of exposure to cosmic radiation with accumulation of radionuclides, and one period of burial during which time the radionuclides decay. In order to evaluate the appropriateness of fluvial sediments for burial dating, both the depositional fabric of the sediment and the relation of the cave passage to other passages (including surface sinks) must be carefully examined to determine whether or not the sediment has been reworked from elsewhere. To check the reproducibility of the dating technique and to check against multiple periods of exposure, duplicate samples were analyzed from sediments deposited at the same elevation but separated by several hundred meters of horizontal passage.

Burial ages are reported with two uncertainties (Table 1); the first is one standard error of analytical uncertainty. The second (parenthetical) uncertainty includes systematic uncertainties in radioactive decay rates, production rates, and initial concentrations (see Anthony and Granger, 2004, 2006a,b for methods and analyses). Analytical uncertainties were used when comparing burial ages between caves in the study area.

Modeling stream power in the Upper Cumberland River basin

The entrenched nature of the Upper Cumberland River, the presence of lithologically-controlled knickpoints and knickzones in the modern river profile, and progressively younger dates of cave passage abandonment moving headward along

Table 1 Cosmogenic nuclide data and burial ages from caves in Upper Cumberland River basin, Tennessee

Cave ID	El. above modern river (m)	Sample weight (g)	[^{26}Al] 10^6 at g^{-1}	[^{10}Be] 10^6 at g^{-1}	[^{26}Al]/^{10}Be	burial age[a] (Ma)
F	43	139.77	0.311 ± 0.015	0.165 ± 0.004	2.31 ± 0.14	1.97 ± 0.10(0.17)
BS	49	72.56	0.379 ± 0.038	0.124 ± 0.009	3.07 ± 0.38	1.66 ± 0.23(0.28)
S	45	121.13	0.262 ± 0.015	0.061 ± 0.006	4.44 ± 0.50	0.89 ± 0.21(0.22)
W	37	79.58	0.188 ± 0.045	0.077 ± 0.005	2.46 ± 0.62	2.15 ± 0.47(0.52)
B	48	101.31	1.12 ± 0.264	0.346 ± 0.012	3.26 ± 0.77	1.45 ± 0.42(0.45)
X	52	138.64	0.207 ± 0.026	0.066 ± 0.014	3.13 ± 0.76	1.64 ± 0.46(0.48)
Z	46	105.45	1.27 ± 0.228	0.481 ± 0.011	2.65 ± 0.48	1.80 ± 0.31(0.36)

[a] Burial age and erosion rate calculated using 1.93 Ma for ^{10}Be meanlife and 1.02 Ma for ^{26}Al meanlife. (See Anthony and Granger (2004, 2006a) for equations and methods.)

correlated cave levels offer compelling evidence of incision by knickpoint migration. We determined the speed of knickpoint migration up the Cumberland River and its tributaries by using a stream power model with drainage area, stream gradient, and absolute time inputs following (2). Assumptions made when applying this model to the Upper Cumberland River basin included; (1) stream power governed incision; (2) discharge was proportional to drainage area; (3) uplift was zero; (4) m,n, and k remain constant along the river profile; (5) the channel was at steady state prior to sudden base level fall; and (6) the modern profile represents the paleoprofile.

Data for river distances and stream gradient were obtained from United States Geological Survey (USGS) 7.5' topographic quadrangles and United States Army Corps of Engineers (USACE) navigation charts. Data for stream drainage area A were gathered from USGS Water Resources streamflow gaging stations, USACE navigation locks, and USGS Water Resources Investigations. Drainage areas in the headwaters of the Caney Fork and Obey River tributaries were measured by planimeter from USGS 7.5' topographic quadrangles.

Stream distance, elevation, gradient, and drainage area for the mainstem Cumberland River and its tributaries were entered into an Excel spreadsheet, with tributaries joined at points along the mainstem. Burial ages representing time of passage abandonment (with uncertainty and one standard deviation) were recorded at points along the tributaries where caves are located. Excel Solver was used to find solutions for the coefficient k and a start date for a single incision event beginning at the confluence of the Cumberland River and the Caney Fork at Carthage, TN and moving at a wave speed c_e up the Cumberland River and its two tributaries, abandoning seven caves (Fig. 5) at known times (Table 1). The exponent m was fixed at values between 0.5 and 3.5 at 0.25 intervals while $n - 1$ was allowed to vary between 0.5 and 3.5 at 0.25 intervals for each value of m. Limits were given as: start time <5 million years; $k \geqslant 1.0 \times 10^{-10}$; and m and $n \geqslant 0$. The start date and k were allowed to vary as functions of each combination of m and $n - 1$ while minimizing the misfit, or chi-squared (χ^2) between modeled time of knickpoint arrival and actual time of cave passage abandonment.

Results

Table 2 shows the results of modeling a stream-power based incision pulse up a network of tributary distances beginning at the Cumberland-Caney Fork confluence at Carthage, TN (Fig. 5). From Carthage the pulse migrated up the Caney Fork-Calfkiller River and up 120 km of mainstem between Carthage and Celina, TN, and continued up the Obey River to the Wolf River and East Fork-Obey River. The model was applied once using dates of cave abandonment in all tributaries, and a second time with the exception of Wolf River and its one associated cave. Although the Wolf River basin is fluviokarstic, it flows as a perennial river through a highly dissected portion of the western margin, where it has developed a wide valley at the same elevation as the Eastern Highland Rim.

When all data were included in the model run, $m = 2.66$ and $n = 3.92$ with an m/n ratio of 0.68 (Table 2). The calculated knickpoint migration rate over the entire tributary network was 0.2—0.53 m/yr. The model could only predict that knickpoint migration along the 120-km Cumberland River mainstem was 6.0 m/yr, as no actual cave data is available on the river between Carthage and Celina.

A second incision pulse was modeled using only the fluviokarst tributaries with ephemeral discharge. Values were

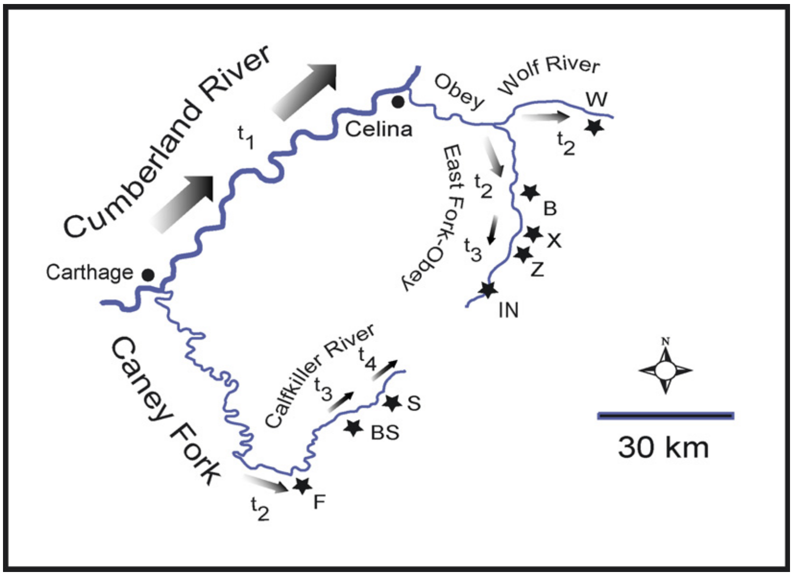

Figure 5 Schematic diagram of migrating knickpoint during Plio-Pleistocene time. Incision pulse originating on the Cumberland River at $t_1 > 2$ million years ago (Ma) migrated up the Caney Fork and Obey Rivers, lowering the local water table and abandoning Foxhole Cave (F) and Wolf River Cave (W) at $t_2 \approx 2$ Ma. At $t_3 \approx 1.6$ Ma, the pulse arrived at Blue Spring Cave (BS) and the caves in the East Fork-Obey (B, X, and Z). Skagnasty Cave (S) was abandoned at $t_4 \approx 0.9$ Ma. Ephemeral reaches located between insurgence (IN) of East Fork-Obey River and B, and upstream of BS in Calfkiller River.

Table 2 Results of a stream-power based knickpoint migration model for Upper Cumberland River fluviokarst tributaries

Cumberland River incision	Distance from start (km)	Cave abandonment (Ma)	Model timing (Ma)	k	m	n	m/n	χ^2	Migration rate (m/yr)
Without Wolf River									
Carthage to Celina (mainstem)	*120*		*2.04*	*2.70E-05*	*1.91*	*2.39*	*0.79*	*0.51*	*4.0*[a]
Carthage to Foxhole (F)	175	**1.97**	1.97						1.1
F to Blue Spring (BS)	215	**1.66**	1.68						0.06
BS to Skagnasty (S)	230	**0.89**	0.89						0.06
Carthage to S	*230*		*0.67*	*2.70E-05*	*1.91*	*2.39*	*0.79*	*0.51*	*0.18*
Celina to Buffalo Cave (B)	238	**1.45**	1.67						0.4
B to Xanadu Cave (X)	250	**1.64**	1.66						0.01
X to Zarathustra's Cave (Z)	251	**1.80**	1.66						0.01
Carthage to Z	*251*		*1.65*	*2.70E-05*	*1.91*	*2.39*	*0.79*	*0.51*	*0.10*
With Wolf River									
Carthage to Celina (mainstem)	*120*		*1.99*	*9.20E-02*	*2.66*	*3.92*	*0.68*	*2.6*	*6.0*[a]
Carthage to Foxhole (F)	175	**1.97**	1.94						0.09
F to Blue Spring (BS)	215	**1.66**	1.86						0.05
BS to Skagnasty (S)	230	**0.89**	0.90						0.05
Carthage to S	*230*		*0.83*	*9.20E-02*	*2.66*	*3.92*	*0.68*	*2.6*	*0.20*
Celina to Buffalo Cave (B)	238	**1.45**	1.71						0.07
B to Xanadu Cave (X)	250	**1.64**	1.71						0.06
X to Zarathustra's Cave (Z)	251	**1.80**	1.70						0.06
Carthage to Z	*251*		*1.70*	*9.20E-02*	*2.66*	*3.92*	*0.68*	*2.6*	*0.53*
Celina to Wolf River Cave (W)		**2.15**	1.97						0.03
Carthage to W	*221*		*1.62*	*9.20E-02*	*2.66*	*3.92*	*0.68*	*2.6*	*0.28*

Wolf River is located within a fluviokarst basin but is perennial.
Bold are radiometric ages. Italics are generated by model from start to end of tributaries.
[a] Predicted value (no cave data for mainstem).

$m = 1.91$ and $n = 2.39$ with an m/n ratio of 0.79 (Table 2). The calculated knickpoint migration rates along the Caney Fork-Calfkiller River and Obey-East Fork Obey River network were 0.18 m/yr and 0.1 m/yr, respectively. Knickpoint migration along the 120-km Cumberland River mainstem was predicted to occur at 4 m/yr.

The smallest misfit ($\chi^2 = 0.51$) between modeled time of knickpoint arrival and known cave abandonment dates was recorded when Wolf River and its one associated cave were excluded. In that model application, the predicted migration rate of 4.0 m/yr for the Cumberland River was within the same order of magnitude compared with 1.57 m/yr for the retreat of Niagara Falls (Tinkler et al., 1994). The migration rates for tributaries were an order of magnitude higher than most other field studies over Plio-Pleistocene timescales (see Tinkler and Wohl, 1998, Table 1) . The ratio m/n generated by the stream power model for Upper Cumberland River fluviokarst was consistent with those determined by previous field and empirical studies. However, the values of m and n were significantly higher when compared to those of other field studies.

Discussion

Can the stream power law be used to model incision by knickpoint migration in fluviokarst basins? The results of this study indicate that it can, providing that higher exponential values for m and n are accepted given the differences between fluviokarst and non-karst channels. The most obvious difference between fluviokarst streams and perennial streams is the division of discharge between the surface and the underlying aquifer in fluviokarst reaches. Since discharge translates into power available for deepening river channels, the values for m and n may be forced higher in fluviokarst. The stream power law relies heavily on the substitution of discharge for drainage area, with discharge values equivalent to the mean annual discharge at stream gauges. Because fluviokarst does not have a linear area-to-discharge relationship, the behavior of channel incision and knickpoint migration may differ substantially from that of non-karst channels due to several distinct but closely related factors.

In fluviokarst basins, as drainage area is increased downstream, discharge in the stream channel also increases but abruptly falls to zero and remains at zero while the drainage area continues to increase. This is due to loss of surface discharge into the underlying karst aquifer at one or more insurgences. Fluviokarst basins may also deliver to the downstream perennial reaches a larger discharge Q per unit area of basin due to a decrease in evapotranspiration, and contribution from stored water in the epikarst, or subcutaneous zone (Hess and White, 1989; Ford and Williams, 1989). This nonlinear relationship between drainage area and discharge is one indication that the drainage area exponent m in the stream power law may be higher in fluviokarst.

The non-conventional relationship between drainage area and discharge can be seen in slope-area plots of fluviokarst channels. In a classic morphometric study of stream profiles, Hack (1957) measured stream length, width, channel gradient, and basin area along several rivers originating on eastern slopes of the Appalachian Mountains and flowing across the Piedmont and Coastal Plain of Maryland and Virginia. Plots of channel gradient versus drainage area on logarithmic axes show that channel gradient decreases as a power law of drainage area. However, the slope θ of the line is steeper for streams originating in the limestone valleys of western Virginia in comparison with clastic bedrock, showing a stronger variance between drainage area and channel slope in the fluviokarst reaches.

The similarity of slope-area plots between our fluviokarstic study area in the Upper Cumberland River basin and the fluviokarstic stream of Hack (1957) (Fig. 6) suggests that there may be an underlying cause that is acting in the evolution of fluviokarst river profiles. (The exception was Wolf River, which is similar to the perennial streams of Hack's study, although much smaller in drainage area.) The stronger variance between drainage area and channel slope for fluviokarst indicates that the area exponent m may be higher. We have not yet developed a working theory to explain this aspect of fluviokarst evolution. However, if we assume that channel incision is still governed by some form of the stream power law, then we can use the slope-area plot to calculate a plausible relationship between discharge and drainage area. A more general form of the stream power law may take the form:

$$E = k(A^{qm}S^{n})^{a} \tag{3}$$

where q is an exponent relating discharge to drainage area A and a is an exponent reflecting the dependence of erosion on basal shear stress. In non-karst watersheds in the southeastern United States, $q \approx 1$, permitting drainage area to be used in place of discharge (Schumm, 1956; Hack, 1957; Brush, 1961). However, we may expect q to be substantially larger than one for fluviokarst basins.

The presence of large horizontal caves is good evidence that stream profiles in the fluviokarst portions of the Upper Cumberland River basin are at long-term steady state. This is because formation of large cave passages in this area requires a long period of discharge ($\approx$0.5 million years) at the same river elevation (Anthony and Granger, 2004). We may therefore reasonably solve for q using the steady-state solution of (3) for:

$$S = kA^{qm/n} \tag{4}$$

If we assume from theory that m/n is approximately 0.7 (Whipple and Tucker, 1999), then the slope θ of the slope-area plot can be used to solve for q. Doing so for the Upper Cumberland River suggests an approximate value of q = 3–4. If we further assume from both theoretical and field studies that knickpoint speed is proportional to discharge, then we can predict that:

$$E = kA^{q} \tag{5}$$

Our prediction in (5) is built upon a series of assumptions that we cannot necessarily justify. We do not know from empirical evidence that discharge in fluviokarst follows a power-law relationship with drainage area. We do not know of any systematic study of discharge/area relationships in fluviokarst streams with which to test this assumption, nor is it clear what the dominant (channel-shaping) discharge in a fluviokarst stream would be. Nonetheless, the steady-state profile of fluviokarst channels suggests a highly nonlinear relationship between incision rate and drainage area.

Finally, another key difference between fluviokarst channels and non-karst may lie in channel width. Generally speaking, downstream changes in channel width are widely considered to vary with the square root of drainage area (Leopold and Miller, 1956; Leopold et al., 1964). In the hydraulic geometry literature, the width of natural channels commonly varies as a function of discharge according to

$$W = k_{w}Q^{b} \tag{6}$$

where W is channel width, Q is discharge, k_w and b are constants, and b is typically 0.5 (Yalin, 1992). In the fluviokarst reaches of the Upper Cumberland River study area, preliminary measurements between the insurgence of the East Fork-Obey River and Buffalo Cave (Table 3) show channel width remains steady as drainage area increases. Translating (6) into the stream power law, if channel width does not increase with downstream distance, perhaps the b exponent for discharge needs to increase more than the constant 0.5 value given. The invariant nature of channel width versus drainage area (discharge) in fluviokarst is another indication of the need for higher values of m.

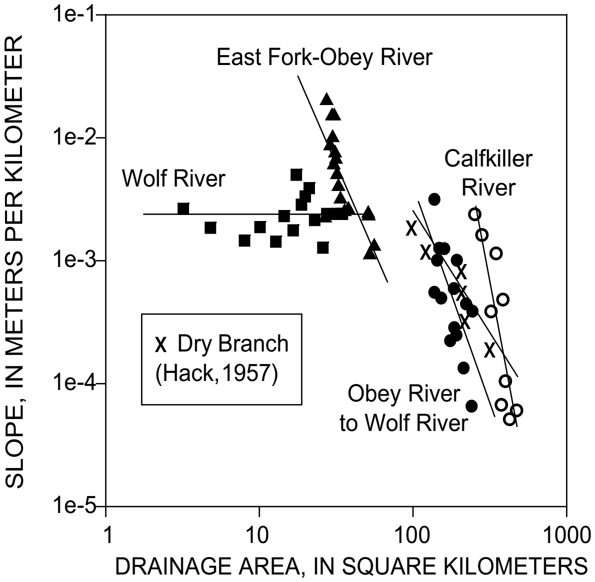

Figure 6 Logarithmic graph showing the relation between drainage area and channel slope for Upper Cumberland River fluviokarst tributaries in the study area. Wolf River does not show a strong variance because of its perennial nature. Included for comparison are data from Hack (1957) for Dry Branch, a fluviokarstic reach in western Virginia.

Table 3 Channel width vs. drainage area for East Fork-Obey River fluviokarst

Points along East Fork-Obey River	Drainage area (km^2)	Channel width (m)
Insurgence	120	31.2
Big Laurel	137	31.3
Sandy Branch	172	38.0
Pratt Branch	315	32.2
Lint's Cove	358	37.4
Buffalo Cave	534	38.2

Stream width is nearly invariant for increasing drainage area.

Conclusions

The stream power law relies heavily on relationships between discharge, drainage area, and stream gradient. For fluviokarstic streams, these relationships may be different than non-fluviokarst. In this study, known dates of river incision at seven cave locations were used to model the passage of a knickpoint in the Upper Cumberland River basin as a response to disruption of steady-state incision, according to the stream power law. The model was applied once to all fluviokarstic tributaries in the study, and once with a perennial stream (Wolf River) removed from the dataset. This study suggests that the stream power model for bedrock incision does explain the migration of a knickpoint in fluviokarst reaches provided that higher values for m and n are accepted. Results (excluding Wolf River) indicate that:

- knickpoint migration rates along fluviokarst tributaries of the Upper Cumberland River were 0.1–0.18 m/yr during the Late Pliocene-Early Pleistocene;
- the ratio $m/n = 0.79$ for fluviokarst tributaries is consistent with previously published parameters in non-karst basins; and
- stream power constants $m = 1.91$ and $n = 2.39$ for fluviokarst tributaries are twice as high as those previously reported for non-karst basins.

Discharge in fluviokarst is divided between surface channels and subsurface conduits, and channel and basin geometries are different when compared to non-karst basins. There is little doubt that modeling the migration of knickpoints in fluviokarst should take these differences into consideration. The presence of caves in similar hydrogeologic settings within the unglaciated Ohio River basin coupled with the ability to date fluvial cave sediments offers a unique opportunity to continue this investigation of Appalachian landscape response to Plio-Pleistocene changes in base-level.

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